

Wanting, Internal Constitutions, and Self-Authorship in Finite Husserlian Monads

From proleptic desire to local submonads and diachronic authorial identity

A self-contained research note in the monadological program

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This paper systematizes an exploratory formal dialogue. The finite-model results are theorems relative to the definitions and explicitly stated nondegeneracy assumptions. Claims connecting formal centered structures to phenomenal subjectivity, or descriptive monadic structure to normative standing, remain bridge principles rather than consequences of finite-state dynamics alone.

Abstract

We develop a finite formal theory of wanting inside Husserlian monads and follow its consequences through proleptic desire, higher-order endorsement, internal social choice, constitutional standing, local submonads, self-consciousness, self-authorship, and diachronic personal identity. A want is separated into desire, goal, intention, endorsement, and reflective stability, yielding thirteen logically consistent desiderative profiles. Proleptic wanting is represented by refinement of partial noemata rather than discovery of a hidden complete preference. Higher-order wants evaluate possible transformations of lower-order wants; conflicts among such endorsements motivate an internal political theory with constitutions, protected standing, and Arrow-style impossibility. A seven-state model is shown sufficient and, under operational witness conditions, necessary for a pluralist, corrigible, self-grounding internal constitution.

The later part of the paper asks when an internal reason-source is more than a metaphorical constituent and becomes a formal local subject. Reason-Source Internalization (RSI) gives derivative standing to independently reason-bearing internal structures, while Centered Monadic Closure (CMC) characterizes a local submonad as a proper centered subsystem closed under the constitutive operations of monadic intentional life. In a dependent intentional-bundle presentation, CMC has an irredundant weak basis consisting of centering, noetic closure, noematic closure, retention, and protention. Under generator faithfulness and joint applicability, the smallest genuine CMC nucleus is a four-state refinement square $C2 \times C2$; a five-state nucleus is minimal for noncommuting path-dependent refinement. The four-state nucleus can already support structural self-consciousness, proleptic self-representation, and structural self-authorship without increasing the state count; matched causal witnesses raise the minimum to five. Finally, we define diachronically reciprocal self-authorship and show that repeated reciprocal transfers of authorship preserve a thin authorial kernel: voice, evaluative integrity, appeal, provenance, and the competence to author again. This suggests a conception of personal continuity not as persistence of preferences but as continuity of reciprocally transferred authorship.

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1. Status and antecedents

The present work belongs to a sequence of experimental formal studies in which Husserlian monads are represented as finite intentional systems rather than as utility-maximizing points. The antecedent manuscripts Possible Worlds of Interacting Husserlian Monads and Minimal Finite Models of Diachronic and Self-Conscious Overminds introduced finite monadic worlds, interaction, diachronic synthesis, and higher-order collective structures [1,2]. A later CEV note developed reflective path dependence, holonomy, least-grounded self-application, standing, and endogenous social choice [3]. A companion PhilLean manuscript supplied a microscopic calculus of wanting, noesis/noema, refinement, higher-order intentional structure, and finite countermodels [4].

The methodology here is deliberately conditional. We distinguish four levels of claim. First are definitions, whose consequences may be elementary once the formal vocabulary is fixed. Second are finite-model theorems and lower bounds under explicit operational assumptions. Third are representational choices, such as treating noeses as a dependent family over noemata. Fourth are substantive bridge principles, such as transperspectival validity or the claim that a formally centered closed intentional subsystem realizes an actual phenomenal subject. This separation is essential: no amount of finite-state combinatorics by itself proves a complete theory of consciousness or ethics.

A recurring methodological theme is that state cardinality and intentional organization are different resources. Several capacities below cost no additional states because they can be represented as richer relations, noemata, or higher-order acts carried by already necessary states. Conversely, a small state increase is sometimes forced when a capacity must be demonstrated by a matched counterfactual rather than merely stipulated structurally.

2. Finite desiderative Husserlian monads

A finite desiderative Husserlian monad (FDHM) is represented schematically by

$$M = (S, A, \rightarrow, O, L, W, \Pi, R, \rho).$$

Here S is a finite set of states; A is a set of external and internal acts; $\rightarrow$ is a transition relation; $O(s)$ is the current intentional/phenomenal presentation; L_s is the representational language available at s ; W_s is the set of active want tokens; $\Pi(s)$ is an executive selector; R is a family of reflective or self-modifying acts; and ρ transports wants across development. The centered-history horizon H_s records the relevant intentional possibilities from the current center.

A want may be represented as

$$w = (\alpha_w, \leq_w, \text{Sat}_w, \gamma_w, C_w),$$

where α_w indicates activation, $\leq_w$ is an incomplete evaluative preorder over centered histories, Sat_w is a satisfaction region, γ_w is an optional explicit representation of the goal, and C_w records causal coupling to action selection. The point is not that this tuple exhausts phenomenology, but that it separates distinct functional and intentional roles that ordinary language often compresses into the single verb want.

2.1 Five desiderative predicates

At a state s , define five predicates on a want w .

Desire $D(w,s)$: the want is active and makes at least one strict evaluative distinction among possible histories.

$$D(w,s) \text{ iff } \alpha_w(s)=1 \text{ and exists } h+, h- \text{ with } h+ >_w h-.$$

Goal $G(w,s)$: the desire has an explicit target representation γ_w whose denotation is its satisfaction region.

$$G(w,s) \text{ iff } D(w,s) \text{ and exists } \gamma_w \text{ in } L_s \text{ with } [[\gamma_w]] = \text{Sat}_w.$$

Intention $I(w,s)$: the goal representation is causally coupled to executive action selection, in the interventionist sense that changing the target representation changes action toward histories ranked better by w .

$$I(w,s) \text{ iff } G(w,s) \text{ and } \text{Ctrl}(\gamma_w, \Pi, s).$$

Endorsement $E(w,s)$: a distinct higher-order want prefers matched continuations in which w persists over continuations in which it disappears. Endorsement is therefore not identical to wanting, goal representation, or executive commitment.

Reflective stability $R(w,s)$: along admissible reflective development, the transported want remains noematically equivalent to its predecessor, with at least one nontrivial reflective transition. This is a diachronic identity condition, not mere current activation.

The only immediate definitional implications are

$$I \Rightarrow G \Rightarrow D, \quad E \Rightarrow D, \quad R \Rightarrow D.$$

3. The desiderative profile theorem

Define the desiderative profile

$$\tau(w,s) = (D,G,I,E,R) \text{ in } \{0,1\}^5.$$

The constraints above leave a chain of three possibilities in the D-G-I coordinate: no desire; desire without explicit goal; goal without intention; or intention. Conditional on positive desire, the D-G-I component has three levels, while endorsement and reflective stability are independent Boolean coordinates.

For positive desire the profile space is therefore isomorphic to

$$T+ \simeq C^3 \times B^2 \times B^2,$$

with twelve profiles, plus a single all-zero no-desire profile.

13-State Desiderative Separation Theorem. Every one of the thirteen profiles consistent with $I \Rightarrow G \Rightarrow D$, $E \Rightarrow D$, and $R \Rightarrow D$ is realizable by a finite FDHM. Any single monad that internally distinguishes all consistent profiles must therefore possess at least thirteen conatively distinguishable states; thirteen suffice.

Proof. Realizability is by independent control of explicit goal representation, executive coupling, higher-order endorsement, and reflective transport over the three-level D-G-I chain. The lower bound follows because a state-determined profile map distinguishing all profiles requires at least one distinct state per profile. This is a minimum for simultaneous logical separation of all species of wanting, not a minimum for one fully developed want.

The philosophical interpretation is useful: D records axiological orientation; G thematic noematic articulation; I volitional fiat; E reflexive intentionality toward a first-order noesis; and R diachronic synthesis of conative identity. The separation theorem prevents us from silently identifying these phenomena.

4. Proleptic wanting as noematic refinement

A central problem is wanting whose object is not yet fully constituted. Rather than positing a hidden complete preference that reflection merely discovers, we model the object itself as partially open.

Fix a finite horizon H and a set Θ of complete noematic articulations. Each θ in Θ induces a total preorder $\succsim_\theta$ on H . A partial noema is a nonempty set $P \subseteq \Theta$. Its settled comparison is the intersection of the comparisons shared by all remaining completions:

$$h \succsim_P k \text{ iff for every } \theta \text{ in } P, h \succsim_\theta k.$$

Refinement is reverse inclusion: P' refines P when $P' \subseteq P$. Hence refinement is monotone in settled information:

$$P' \subseteq P \Rightarrow \succsim_P \subseteq \succsim_{P'}.$$

A refinement can settle a comparison that was previously unresolved but cannot reverse a comparison already settled by every completion. A reflective lineage is therefore

$$P_0 \supseteq P_1 \supseteq P_2 \supseteq \dots$$

Weak prolepsis requires an initially non-singleton P and at least one strict refinement. Genuine prolepsis requires branching into substantively different legitimate refinements. A canonical witness has two descendants P1 and P2 such that

$$h >_{\{P1\}} k, \text{ but } k >_{\{P2\}} h.$$

The structure is therefore not mere uncertainty about a privileged hidden θ^* . In self-discovery, θ^* is already fixed and the monad is ignorant of it. In genuine prolepsis, several future articulations are legitimate continuations of the present wanting.

Minimal Genuine Prolepsis Theorem. Under the assumption that each state determines its current partial noema, strict nonbranching refinement requires two states, while genuine branching prolepsis requires three: one root and two distinct refinement descendants.

The natural object is a refinement tree T_w , and mature wanting need not converge to a singleton. Stability therefore separates into fixed-content stability and lineage stability. Fixed-content stability implies lineage stability, but a want may remain the same developing want while its legitimate noema becomes more determinate.

5. Higher-order endorsement and conflict

A proleptic want can itself become the object of higher-order evaluation. At a partial noema P, let $Dev(P)$ contain staying as one is, abandoning the want, and every admissible strict refinement $P \rightarrow P'$. A higher-order want v can compare developmental histories rather than worldly outcomes.

Let Ξ_w be a finite set of complete developmental noemata. A partial higher-order noema Q subseq Ξ_w induces a robust comparison

$$\delta_1 \geq_Q \delta_2 \text{ iff for every } \xi \text{ in } Q, \delta_1 \geq_{\xi} \delta_2.$$

For a refinement edge e, define permission, positive endorsement, and rejection relative to staying:

$Perm_Q(e)$: every completion ranks e at least as high as stay.

$End_Q(e)$: every completion ranks e strictly above stay.

$Rej_Q(e)$: every completion ranks stay strictly above e.

This separates actualization from endorsement. A development may occur without being endorsed; an endorsed development may fail to occur. A causal notion of endorsement additionally requires that intervention on the higher-order attitude change the probability of the lower-order development.

Minimal Causal Endorsement Theorem. Genuine branching prolepsis plus internally variable endorsement and a matched counterfactual requires four states under the state-distinct-noema convention. Purely semantic endorsement can be represented within the three-state branching model.

5.1 Conflict among higher-order wants

Let $V = \{v_1, \dots, v_n\}$ be active higher-order wants evaluating the same developmental agenda. Two wants robustly conflict when they reverse at least one pair of options. Top sets and acceptable sets distinguish consensus, negotiable conflict, and hard conflict. This leads naturally to a reflective constitution C whose choice correspondence is

$$\chi_C : \text{profiles of higher-order preferences} \rightarrow \text{nonempty subsets of } Dev(P).$$

Different architectures include compromise, multiplexing, lexicographic dominance, veto, annihilation, totalization, and rights-constrained arbitration. A three-option Condorcet cycle already appears with three higher-order wants having cyclic rankings, so internal deliberation inherits ordinary social-choice pathologies.

6. Internal constitutions and monadic political philosophy

Once several higher-order wants are treated as separately reason-bearing constituents, a single monad acquires an internal analogue of political structure. The analogy is formal rather than anthropomorphic: constituents need not be

persons. What matters is that distinct higher-order reason-sources can disagree about self-development and that some rule aggregates, negotiates, or adjudicates those reasons.

Let $X_s = \text{Dev}(P_s)$ and let $R_s = (\geq_1, \dots, \geq_n)$ be the profile of internal higher-order preferences. A constitution maps the profile to one or more authorized developmental moves. If the constitution instead constructs a single complete transitive collective ranking over at least three alternatives, classical Arrow conditions immediately apply [5]. Thus unrestricted domain, unanimity, IIA, completeness/transitivity, and non-dictatorship cannot all be retained.

This motivates relaxing IIA: the availability of alternative possible selves can legitimately affect how transformations are evaluated. It also motivates rights that protect internal standing rather than outcomes.

6.1 Four constitutional rights

For a protected internal constituency K , four structural rights were isolated:

- Survival: a protected reason-source retains a transported descendant unless it validly relinquishes standing.
- Voice: the descendant remains constitutionally non-null; changing its reasons can still matter somewhere in the admissible profile domain.
- Anti-colonization: its evaluative content is not externally overwritten and then falsely treated as its own endorsement.
- Appeal: defeat does not permanently remove all future reflective routes by which the issue could be reopened.

These are structural rights, not rights to get what a constituent wants. The central distinction is

conative rights $\neq$ conative preferences.

If every protected want has an absolute veto over every substantive move it ranks below staying, hard conflict can produce paralysis. The resulting lesson is: protect the capacity to want and to remain a source of reasons, not every object that is wanted.

6.2 An anapartistic leximin constitution

Let $S_K(P)$ be the moves preserving survival, voice, anti-colonization, and appeal for every protected source. Rank each safe option by each constituent, translate ranks into dissatisfaction values, sort each option vector from worst to best, and lexicographically minimize that sorted vector. This yields a finite rights-constrained leximin choice rule. It is Pareto-respecting on the safe set and has no intrinsic dictator when at least two protected wants matter, while deliberately violating IIA.

7. The seven-state constitutional monad

A useful benchmark is the smallest operational finite monad that witnesses pluralist internal constitutional agency rather than merely stipulating it counterfactually. Require two protected higher-order wants, genuine conflict, common-base local pivotality of both wants, decisiveness, Pareto respect, non-dictatorship, distinct refinement outcomes, defeat with survival and appeal, authorized constitutional amendment, and a matched constitution-only causal effect.

Use three developmental alternatives a, b, c . At a central deliberative state q_0 let

$$v_1: a > b > c, \quad v_2: b > c > a.$$

A Borda-like constitution C_0 chooses b . At q_1 change only v_1 so that c becomes the collective choice; at q_2 change only v_2 so that a becomes the collective choice. This witnesses local pivotality of both constituents and also non-dictatorship, because at q_0 v_1 does not get its top choice and at q_1 v_2 does not get its top choice.

Let r_a, r_b, r_c be three distinct outcome/refinement states. Then

$$q_0 \rightarrow r_b, \quad q_1 \rightarrow r_c, \quad q_2 \rightarrow r_a.$$

Appeal can be represented by reflective transitions from each r_x back into deliberation without adding states. Add q_0 prime with the same current noema and internal preference profile as q_0 but a different constitution C_1 , choosing a . An authorized amendment $q_0 \rightarrow q_0$ prime then witnesses causal constitutional variation.

Seven-State Constitutional Theorem. Under the operational witness requirements above, seven states are necessary and sufficient: $q_0, q_1, q_2, q_0 \text{ prime}, r_a, r_b, r_c$.

Proof. Three deliberative states are required to witness the local pivotality of two distinct constituents around a common base. Three distinct refinement outcomes are disjoint from those deliberative states, giving six. A constitution-only matched variant cannot be any of the six because its current noema and preference profile match q_0 while its constitutional rule differs and yields a different outcome. Hence at least seven; the construction realizes seven.

A weaker structural model can be smaller if non-null voice is stipulated over uninstantiated counterfactual profiles. Seven is therefore the preferred operational minimum, not an unconditional metaphysical state count.

8. Self-grounding, standing, and first-person validity

A constitution can itself become an object of the same higher-order choice rule by making the rule agenda-polymorphic: it evaluates ordinary alternatives and candidate constitutions alike. For a constitutional profile $R^A C$, define

$$F_{\{R^A C\}}(C) = \chi_C(R^A C).$$

A reflective constitutional equilibrium is a pair (C, s) such that $F_s(C) = C$. Unconditional self-ratification at every admissible profile would destroy corrigibility, but situated self-grounding is compatible with amendment: a constitution may ratify itself at one state and be replaced at another. The seven-state model can realize both without an eighth state, using a ratifying self-loop in one deliberative state and the existing amendment transition elsewhere.

Self-Grounding-Corrigibility Result. A single-valued constitution that ratifies itself for every admissible constitutional profile is not corrigible. Situated self-grounding avoids the contradiction: C may satisfy $F_s(C) = C$ at one state while some reachable t satisfies $F_t(C) = C \text{ prime} \neq C$.

The earlier CEV work used least fixed points to distinguish grounded self-application from merely self-supporting cycles [3,6]. That lesson remains important: descriptive self-reference can be cyclic, but normative authority should not be manufactured by an unsupported justificatory loop.

8.1 Diachronic Reciprocal Standing

Define the standing vector of an internal reason-source v as

$$\text{Stand}(v, s) = (E_v, N_v, I_v, A_v),$$

where E is continued lineage existence, N non-null voice, I evaluative identity/non-colonization, and A appeal. Diachronic Reciprocal Standing (DRS) requires that every protected reason-source preserve standing through every permissible move unless it validly authorizes release:

$$\text{not Release}_v(e) \Rightarrow \text{Stand}(\rho_e(v), s \text{ prime}).$$

Under the definitions, DRS is essentially equivalent to the conjunction of survival, voice, anti-colonization, and appeal.

8.2 First-Person Standing

Two previously separate ideas can be unified. Self-standing says a reason-source, from its own standpoint, has a defeasible claim not to be involuntarily erased as a source of reasons. Symmetry of standing says that the constitutional significance of this claim cannot depend merely on whether the token is named v_1 rather than v_2 . Represent a constituent as a pointed constitutional state $(s; v)$, and identify pointed states up to structure-preserving isomorphism.

First-Person Standing (FPS) is then

$$\text{Src}(v, s) \text{ and not Release}_v(e) \Rightarrow \text{Stand}(\rho_e(v), s \text{ prime}),$$

with the predicate understood on the pointed isomorphism class. FPS yields both self-standing and permutation-invariant symmetry; conversely those two conditions determine FPS on pointed isomorphism classes. Hence FPS $\rightarrow$ DRS $\rightarrow$ the four constitutional rights.

9. Can first-person standing be derived from monadic ontology?

The next audit asked whether FPS follows from the ontology of Husserlian monads itself. The answer is nuanced. Descriptive monadic ontology supplies perspectival centers and legitimate recenterings, but by itself does not prohibit a haecceitistic constitution that privileges the center called me. A fully informed egoist can recognize another center as equally real while denying that its reasons have equal collective authority; no formal contradiction follows. The CEV paper therefore isolates a residual bridge principle, Transperspectival Validity (TV): a genuinely collective norm must transform covariantly under legitimate recenterings that preserve all grounded relevant structure [3].

$$F(\sigma r; \sigma W) = \sigma F(r; W).$$

With normative interiority and TV, synchronic non-haecceitistic standing follows for whole monads. A Sufficient Normative Difference principle then transports standing diachronically when the relevant first-person structure persists. A non-manufactured-standing-loss principle is needed to block the maneuver erase the subject, therefore its claims no longer count.

However, the internal political theory concerns wants or reason-sources within one monad, not whole monads. A want is normally a noetic/desiderative structure of a subject, not automatically another subject. This produces the central gap that motivated Reason-Source Internalization.

10. Reason-Source Internalization

Reason-Source Internalization (RSI) asks when an internal higher-order want deserves independent constitutional representation without assuming that every motive is a miniature person. The solution is to define a Local Reason-Source (LRS) by six conditions.

1. Intentional individuation: the source is an articulated intentional act with its own noematic content, not merely a causal parameter.
2. Self-attribution: the host represents the source as part of its own intentional economy; endorsement is not required.
3. Evaluative reasonhood: the source induces genuine comparisons among developmental possibilities available to reflection.
4. Counterfactual separability: admissible states can vary the source while holding the other relevant sources fixed.
5. Diachronic trackability: transport rho identifies descendants of the source through reflection.
6. Reflective addressability: other acts can represent, endorse, reject, refine, or deliberate about the source.

To avoid internal vote stuffing, reason-sources are individuated by an equivalence relation. Two syntactic want tokens are equivalent when they generate the same reasons in every admissible context, undergo corresponding transports, have the same reflective relations, and cannot vary independently. Constitutional constituency is therefore a quotient of LRS tokens, not a raw count of representations.

10.1 Derivative RSI

A whole-self constitution that purports to represent the monad's reflective agency must not silently erase an independently reason-bearing internal distinction. Semantic branch completeness supplies a derivation: if two matched configurations differ only in v and yield different grounded constitutional conclusions, collapsing them by deleting v destroys a decision-relevant distinction. Hence an LRS equivalence class must remain in the constitution's domain.

Derivative RSI therefore grants constitutional standing to an LRS as a constituent of the host without claiming that the LRS is independently conscious. This is standing through faithful self-government rather than intrinsic patienthood.

10.2 Three internal statuses

Status	Criterion	Standing
Motive	Causal desiderative influence without independent reason-source organization	No separate constitutional standing follows
Constituent reason-source	Satisfies LRS	Derivative constitutional standing
Local submonad	LRS plus the structural conditions for a centered self-synthesizing intentional subsystem	Candidate intrinsic first-person standing

The remaining task is therefore to identify the threshold between an LRS and a genuine local submonad.

11. The local-submonad threshold

The first attempt decomposed the threshold into Perspectival Closure (PC) and Endogenous Temporal Synthesis (ETS). PC says that the subsystem has a local intentional pole and a horizon whose relevant noetic/noematic interpretation closes around that pole. ETS says that retention and protention synthesize successive phases from within the subsystem rather than merely being tracked externally by the host.

Closer analysis showed that PC and ETS share a common ingredient, centering. Write

$$PC = C \text{ and } H_0, \quad ETS = C \text{ and } T_0,$$

where C is local centering, H_0 semantic/noematic horizon closure without presupposing centering, and T_0 temporal closure without presupposing centering. The weak theory admits PC without ETS and ETS without PC: a sequence of instantaneous local perspectives can lack endogenous temporal unity, while a temporally self-maintaining proto-center can remain semantically parasitic on the host. Thus neither implication is derivable without additional thickness principles.

Rather than add separate axioms saying horizon closure entails temporal synthesis or vice versa, the dialogue found a deeper common condition: Centered Monadic Closure.

12. Centered Monadic Closure

Let Σ_{mon} be the constitutive operation signature of monadic intentional life. It includes noetic and noematic operations, retention and protention, and the relevant higher-order/evaluative transports. For an LRS v , let G_v be its candidate intentional subsystem. Centered Monadic Closure (CMC) requires a local pole c_v such that every endogenous monadic operation applicable to arguments in G_v has its result in G_v , and every member of G_v belongs to the intentional component organized around c_v .

CMC is closure, not isolation. Inputs from the host or environment may enter through interfaces. What must be internal is the constitutive completion of meaning, refinement, retention, protention, and locally generated higher-order relations. Nor does CMC require different developmental operations to commute. If semantic and temporal paths lead to different internal endpoints, the resulting path dependence belongs to the local subsystem itself.

This is strongly suggested by the intentional-bundle formalism of the PhilLean paper, where noeses form dependent fibers over noemata and noetic/noematic commutation is an additional naturality condition rather than a basic axiom [4]. The same paper explicitly proposes that an overmind should integrate refinement structures while satisfying coherence conditions, rather than merely pooling intentional contents [4].

Monadic Integration Equivalence Theorem. Under a Signature Generation Hypothesis stating that the monadic operations not already supplied by LRS are generated by semantic/horizon and temporal operation families, $LRS + CMC$ is equivalent to $LRS + C + H_0 + T_0$.

Proof. CMC immediately yields centering and closure under the semantic and temporal generators.

Conversely, closure under the generating families and the residual LRS operations is preserved under finite composition, giving closure under the entire generated monadic signature.

This gives the compact formal threshold

$$\text{FormalSubMonad}(v) \text{ iff } LRS(v) \text{ and } CMC(v),$$

subject to the structuralist definition of formal monadhood. A further bridge principle is needed if one wants to infer phenomenal subjectivity from formal centered closure. Call this Centered-Structure Realization (CSR): every realized centered temporally synthesized intentional substructure instantiates an actual first-person center. CSR is not proved by the finite formalism.

12.1 The irredundant weak basis of CMC

In the dependent intentional-bundle representation, the most natural weak basis is

$$\text{Sigma_CMC} = \{C, N, M, R, P\},$$

where C is centering; N closure under endogenous noetic refinement; M closure under noematic refinement/transport; R retention closure; and P protention closure. Noesis already carries its noema as a dependent type index, so an explicit noema-extraction map η need not be counted separately. In a flat representation with explicit η , the basis rank becomes six.

Each of C,N,M,R,P is independent of the other four in the weak theory: one can construct a finite subsystem with a local failure of exactly that operation family and internal closure of the others. Thus there is no general eighty-percent integration theorem. Full CMC is a qualitative closure condition, not a scalar threshold.

13. Minimal CMC nuclei

Raw CMC admits trivial one-state self-loop models, so a meaningful minimality theorem requires generator faithfulness: noetic and noematic refinement must be separately observable dimensions rather than two names for one transition. Add joint applicability: after refining either dimension, the other remains nontrivially applicable.

Begin with a seed $x00$. Strict noetic refinement gives $x10$; strict noematic refinement gives $x01$, with $x10 \neq x01$ by generator faithfulness. Joint applicability requires a state refined along both dimensions, $x11$. In the flat/commuting case the two paths meet:

$$N(x00)=x10, \quad M(x00)=x01, \quad M(x10)=N(x01)=x11.$$

The resulting square is the four-element product $C2 \times C2$:

$$\begin{array}{ccc} x00 & \xrightarrow{\quad N \quad} & x10 \\ | & & | \\ M & & M \\ | & & | \\ \downarrow & & \downarrow \\ x01 & \xrightarrow{\quad N \quad} & x11 \end{array}$$

Retention and protention can be represented on the same four states, for example by treating the root as protending the two one-step refinements and the top as retaining both immediate predecessors. Centering is a relation organizing the states around one transported pole and therefore costs no fifth state.

Minimal CMC Nucleus Theorem. Under generator faithfulness and joint applicability, the smallest nondegenerate commuting CMC nucleus has four states. A one-state LRS-bearing seed can generate this four-state monadic hull in two semantic closure rounds.

Proper submonadhood requires the hull to be a proper subset of its host. Hence the smallest host containing such a nucleus has five states. The extra host state can also be used as a matched operational witness for independent reason-source relevance.

13.1 Path-dependent nucleus

If noetic and noematic refinement fail to commute, split the upper-right corner:

$$M(N(x00)) = x11^{(NM)}, \quad N(M(x00)) = x11^{(MN)},$$

with distinct endpoints. Then five states are necessary and sufficient for the smallest locally path-dependent CMC nucleus, and a proper host requires at least six. This mirrors the earlier five-noesis noncommutation countermodel in the intentional-refinement work [4].

13.2 Monadic closure hull and nucleation

For an LRS seed and candidate center c , iteratively close under the CMC generators:

$$A_{\{n+1\}} = A_n \text{ union } N(A_n) \text{ union } M(A_n) \text{ union } R(A_n) \text{ union } P(A_n).$$

Because the host is finite, this reaches a least fixed point $\text{Hull_mon}(v,c)$. An LRS nucleates a proper formal local submonad exactly when some candidate center generates a centered, nondegenerate fixed point H strictly smaller than the host. If every closure attempt expands to the whole host, the reason-source remains constitutively parasitic on the host subject.

Closure rank and closure depth are distinct invariants. The weak dependent-bundle basis has rank five, while depth measures the number of rounds needed for a particular seed to reach its fixed point. Neither is simply the number of automaton states.

14. Self-consciousness and proleptic self-consciousness

CMC gives a centered subject-like subsystem, but does not imply reflective self-consciousness. Add Reflexive Reification (RR): the centered CMC whole becomes the intentional object of one of its own noeses. Let Sigma_G be a noema denoting the pointed isomorphism class (G,c) , and let r_G be a self-ascriptive noesis directed at Sigma_G . An Indexical Binding Condition (IBC) requires the represented center to be the very center from which r_G is performed.

$$[[\text{Sigma_G}]] \sim (G,c), \quad \text{subj}(r_G) = \text{representedCenter}(\text{Sigma_G}) = c.$$

Because a state can carry multiple intentional acts and relations, RR can be placed at x11 of the four-state square without adding a fifth state. CMC alone does not imply RR: the same square with no self-noema is a countermodel.

Four-State Structural Self-Consciousness Theorem. Under generator-faithful CMC, structural reflective self-consciousness via RR+IBC requires no additional raw state beyond the four-state nucleus. Four are necessary because CMC already requires four under the nondegeneracy conditions.

If self-consciousness must be causally witnessed, add a matched upper-right state with the same ordinary refinement coordinates but different self-model availability, and require the difference to alter reflective selection. This raises the local minimum to five.

14.1 Proleptic self-consciousness

Static self-consciousness represents what the nucleus is now. Proleptic self-consciousness (PSC) represents what it may become. Associate to every state x a self-noema Sigma_x whose denotation is the pointed space of admissible future developments reachable from x . At the root, the self-model therefore represents both immediate refinement branches and their descendants.

For every actual refinement $e:x \rightarrow y$, require Self-Refinement Naturality

$$e_*^S(\text{Sigma}_x) = \text{Sigma}_y.$$

A Diachronic Indexical Binding Condition further requires the future center represented in Sigma_y to be represented as the transported continuation of the present center, not merely as another future system.

Soundness forbids imaginary future selves; completeness requires every still-live conatively distinct future self to remain represented. Hence a branch-complete self-model cannot falsely merge two path-dependent descendants merely because their coarse current descriptions coincide.

Four-State Proleptic Self-Consciousness Theorem. The four-state CMC refinement square can carry a branch-complete future-self model with self-model transport mirroring the ordinary refinement square. Structural PSC therefore has the same four-state minimum; a matched causal witness raises the minimum to five.

A convenient formalization is coalgebraic:

$$\text{Sigma}_x = (\text{Obs}(x), \{(e, \text{Sigma}_y) : x \xrightarrow{e} y\}).$$

Finite cyclic graphs then represent self-awareness of looping or holonomic futures without an infinite syntactic regress.

15. Self-authorship

A self that merely predicts or prefers its future remains a spectator. Self-authorship begins when the current centered self evaluates and authorizes developmental paths and its authorial judgment has causal control over which path is taken.

Let D_x be the set of admissible self-development paths γ beginning at x , and let $\geq_x^A$ be an authorial preorder over paths. Path ranking is more fundamental than endpoint ranking when refinement is noncommutative: becoming nominally similar through N then M may be intentionally different from becoming through M then N .

An authorial gate

$$\text{Gamma}_x : (\text{Sigma}_x, \geq_x^A) \rightarrow \text{nonempty subsets of } D_x$$

selects the transformations the present self authorizes. Authorial Indexical Binding requires the authorial act to be owned by the very local center represented in the self-model. Causal authorial control requires intervention on the authorial evaluation or authorization, holding the ordinary state fixed, to change the selected self-development.

Strong self-authorship can therefore be summarized as

$$\text{SA} = \text{PSC} + \text{owned authorial evaluation} + \text{authorization} + \text{causal self-direction}.$$

15.1 Provenance and no retrospective fabrication

An authored transformation should preserve an authorial trace recording the alternatives genuinely represented as live, the authorial ranking, the authorization set, and the actualized path. Otherwise the descendant could rewrite its autobiography so that the realized branch appears retrospectively inevitable. Actualization therefore does not license erasure of the counterfactual alternatives that were live at the time of authorship.

Structural self-authorship fits in the four-state square: the root carries the proleptic self-model and an authorial ranking over N and M , while the chosen descendant retains the trace. A matched pair of root states that differ only in authorial evaluation and choose different branches gives the five-state causal witness.

Finite Self-Authorship Minima. Under the established CMC nondegeneracy assumptions, structural self-authorship has minimum four states; operationally witnessed causal self-authorship has minimum five.

15.2 Constitutional self-authorship

Self-authorship need not be dictatorship by one present impulse. If the local self contains several protected internal reason-sources, the authorial gate can itself be an internal constitution aggregating their higher-order judgments over prospective self-transformations. Rights-constrained authorship requires authorized paths to preserve the relevant FPS/DRS standing of those constituents unless they validly relinquish it. This yields constitutional self-authorship: the self as an organized plurality governs its own development.

The authorial rule can itself enter the self-model and be reflectively ratified or revised. Situated self-grounding therefore applies to authorship as it did to constitutions, avoiding an infinite regress of author of the author by self-applicative representation.

16. Diachronically reciprocal self-authorship

Present authorship raises a temporal political problem. The present self exists and can deliberate; merely possible future selves do not yet exist and should not be given ordinary current votes. Yet treating the present self as absolute owner of its descendants turns authorship into temporal dictatorship. The solution is branch-conditional future standing.

For a live path $\gamma: x \rightarrow \dots \rightarrow y$, define prospective standing $P_{\text{Stand}_x(y|\gamma)}$. This does not give y a right to be actualized. It says that conditional on actualizing γ , the resulting descendant must possess the standing appropriate to a genuine later author. Extend the standing vector with provenance:

$$D_{\text{Stand}}(y) = (E_y, N_y, I_y, A_y, P_y).$$

P_y is authenticated access to the history of how y came to be. The authorial trace is inherited by the descendant, allowing it to distinguish legitimate inherited commitments from imposed conditions.

16.1 Ratification versus agreement

Future reciprocity should not require future preference agreement. A descendant may regret the path while judging that its predecessor was entitled to authorize it. Define a retrospective constitutional evaluator

$$J_y(\text{trace}_\gamma) \text{ in } \{\text{ratify, suspend, repudiate}\}.$$

Regret concerns whether y now prefers another outcome or path; repudiation concerns whether y judges the predecessor lacked legitimate authority. Regret and ratification can therefore coexist. This is essential for transformative agency, because otherwise reciprocity would collapse into dynamic preference consistency.

At the present state, prospective ratifiability uses the proleptic self-model to represent how admissible descendant models would assess their own genesis. Robust prospective ratifiability requires that no still-live grounded descendant model constitutionally repudiates the path. Actualization later closes the loop by comparing the represented descendant assessment with the actual descendant's judgment.

16.2 Non-manufactured ratification

A present self must not guarantee future approval by rewriting the future evaluative or constitutional machinery specifically to praise the present act. Such manufactured future ratification is the temporal analogue of colonization. A valid future ratification therefore requires that the path not engineer the descendant's approval in precisely the respect used to justify the path.

16.3 Definition and finite minima

A path is Diachronically Reciprocally Authored (DRA) when it is presently authored, preserves branch-conditional future standing and provenance, is prospectively ratifiable, and does not manufacture the descendant's judgment through colonization. DRA preserves authorship across change rather than preserving substantive preferences.

Transformative Reciprocity Theorem. DRA is compatible with reversal of first-order preference. There can be $x \rightarrow y$ such that $A \succ_x B$, $B \succ_y A$, while y ratifies the predecessor's authority to have chosen the transformation. Hence DRA does not imply preference persistence.

Structural DRA can be represented within the four-state square. For an operational witness that concern for the future standpoint genuinely constrains present authorship, duplicate the root: in one matched root the modeled N-descendant ratifies N ; in the other it repudiates N while the M -branch remains ratifiable. Holding current authorial preference fixed, the authorized branch changes. Thus causal DRA has a five-state minimum under the matched-counterfactual convention.

DRA also blocks unilateral self-enslavement: present authorization alone cannot legitimate a transformation that leaves the resulting self permanently mute, colonized, or without all future reconsiderative standing. Binding commitments remain possible because constraint is not the same thing as extinction of future authorship.

17. The diachronic authorial invariant

Repeated DRA does not yet guarantee that the descendant can author another transformation. A self could receive voice, integrity, appeal, and provenance but lack the self-model or causal control required to govern what it becomes next. To obtain an invariant under repeated reciprocal transfers, add Hereditary Authorial Competence (HAC).

$$U(x) \text{ and not Release}_x^U(\gamma) \Rightarrow U(y),$$

where $U(x)$ means that x possesses proleptic self-consciousness, authorial evaluation, authorization capacity, and causal authorial control. Define Recursively Diachronically Reciprocal Authorship (RDRA) as $DRA+HAC$.

The candidate diachronic kernel initially appears to contain lineage existence E , voice N , evaluative integrity I , appeal A , provenance P , and authorial competence U . But relative to CMC plus correct diachronic indexical binding, authenticated provenance already supplies a continuing lineage. The irredundant weak kernel is therefore

$$K_DA = (N, I, A, P, U).$$

Diachronic Authorial Kernel Theorem. Along every RDRA trajectory with no valid authorial release, the transported properties N, I, A, P , and U hold at every stage. Hence the five-component authorial kernel is forward-invariant.

Proof. Each RDRA edge preserves voice, integrity, appeal, and provenance by DRA. HAC preserves authorial competence. Correct diachronic indexical binding transports the lineage. Induction over the finite or infinite sequence yields invariance at every finite stage.

Each of the five components is independent of the other four in the weak theory. Without N the descendant is constitutionally mute; without I its reasons may be manufactured; without A inherited commitments can become permanently self-sealing; without P it cannot audit how it became what it is; without U it is a final citizen rather than the recipient of transferable authorship.

17.1 Identity as continuity of transferred authorship

This suggests a specifically normative notion of diachronic personal continuity. Write $x \sim_{RA} y$ when x and y are connected by a chain of RDRA transformations along which K_DA is transported without involuntary loss. This need not settle the metaphysics of numerical identity. It identifies a thinner but rigorous relation: continuity of reciprocally transferred authorship.

The invariant need not include any fixed desire, project, personality trait, or first-order preference. A radically transformed descendant can belong to the same authorial lineage if it remains a non-null, non-colonized, historically informed, revisable author with power to participate in what it becomes next.

17.2 Fixed points, recurrence, and holonomy

Let A be the set of states satisfying the authorial kernel. A viability transformer that retains exactly those states whose every RDRA successor remains in the set has a greatest fixed point $\nu \Phi$. Membership in this greatest forward-invariant region characterizes indefinite authorial viability. This is conceptually different from the least fixed points used for normative grounding: viability asks for the largest set closed under future transitions, whereas grounding asks for the smallest set supported by independent grounds.

In a finite monad, every infinite RDRA trajectory eventually enters a recurrent strongly connected component. First-order volition may continue to cycle or exhibit holonomy inside that component while the authorial kernel remains fixed. In this sense the kernel can be the holonomy-invariant part of self-authorship even when substantive values are path-dependent.

A complementary identity intersection is obtained by transporting the theories of all RDRA-reachable descendants back to the present and intersecting them. At the minimal structural level, N, I, A, P, U survive every such future even when no first-order preference proposition does.

18. Synthesis, limits, and research program

The dialogue begins with a microscopic question - what kinds of wanting can a finite Husserlian monad possess? - and ends with a candidate invariant of self-authored personal continuity. The path is cumulative. Separation of desire, goal, intention, endorsement, and reflective stability makes room for proleptic wants whose objects are still being constituted. Higher-order endorsement then turns transformations themselves into objects of wanting. Conflict among such endorsements generates an internal social-choice problem. Rights-constrained constitutions protect reason-sources without guaranteeing outcomes, and self-grounding becomes situated rather than self-entrenching.

The standing problem exposes a boundary in the political metaphor. Whole monads are first-person centers; wants normally are not. RSI therefore distinguishes mere motives, derivative constituent reason-sources, and candidate local submonads. CMC then identifies the formal local-subject threshold as centered closure under the constitutive operations of monadic intentional life. The resulting closure-hull criterion makes submonadhood an organizational fixed-point phenomenon rather than a raw state-count threshold.

The four-state CMC square is the central finite nucleus of the later theory. It is the smallest generator-faithful jointly refinable local structure, yet it can simultaneously support centered semantic and temporal closure, structural self-consciousness, a branch-complete future-self model, and structural self-authorship. Richer capacities often cost no states because they are relational/intentional organization on this square. Additional states are forced when we demand matched causal observability, or when path dependence splits a common refinement endpoint.

Diachronic reciprocity then changes the interpretation of self-authorship. The present self is neither the passive victim of future values nor the absolute proprietor of future selves. It acts more like a trustee transferring an office of authorship. Future selves do not possess present rights to be actualized, but if a branch is chosen, the descendant should inherit a genuine standpoint from which it can understand, reassess, and continue the authorial history. The thin invariant $K_DA=(N,I,A,P,U)$ captures that office.

18.1 What is proved, and what is not

The strongest exact results are finite realizability/lower-bound theorems conditional on the formal definitions and operational witness conventions. The 13-state desiderative separation theorem, 3-state prolepsis minimum, 4-state causal endorsement minimum, 7-state operational constitutional minimum, 4-state CMC nucleus, 5-state noncommuting CMC nucleus, and the structural-versus-causal minima for self-consciousness, PSC, self-authorship, and DRA have this status.

By contrast, the move from a formal centered CMC subsystem to an actual phenomenal subject requires CSR or another consciousness theory. The move from descriptive monadic first-personality to impartial collective standing requires TV or an equivalent normative bridge. The authorial identity kernel is therefore best understood as a theorem about a formally defined mode of continuity, not as a complete theory of metaphysical personal identity.

18.2 Open problems

- Formalize the full theory in a proof assistant or finite model finder, including automatic searches for countermodels to proposed implication reductions.
- Classify CMC closure algebras beyond the commuting $C2 \times C2$ nucleus and compute minimal nuclei with controlled curvature or holonomy.
- Determine whether empirically motivated theories of consciousness support CSR, weaken it, or require further structure beyond CMC.
- Develop a theory of overlapping local submonads, where centered hulls share states or intentional acts without collapsing into one subject.
- Analyze stochastic and partially observed versions of prospective future standing and ratifiability.
- Study authorial release: identify conditions under which a present self can validly relinquish future authorial competence without temporal domination.
- Connect the authorial kernel to richer accounts of commitment, promise, responsibility, memory, and transformative experience.
- Investigate whether the five-component authorial kernel can itself be compressed by a deeper structural principle without losing the distinctions that the countermodels expose.

Appendix A. Main finite minima

Structure/capacity	Minimum	Key assumptions
All consistent desiderative profiles	13 states	State-determined profile; distinguish all D/G/I/E/R profiles
Strict nonbranching prolepsis	2 states	State-determined partial noema
Genuine branching prolepsis	3 states	Root plus two distinct legitimate refinements

Causal higher-order endorsement	4 states	Branching plus matched endorsement counterfactual
Operational pluralist constitutional monad	7 states	Two locally pivotal sources, 3 outcomes, appeal, amendment, constitution-only causal witness
CMC nucleus, commuting	4 states	Generator faithfulness + joint applicability
Proper host for 4-state CMC nucleus	5 states	Nucleus is a proper subset
CMC nucleus, noncommuting	5 states	Two distinct path-order endpoints
Proper host for noncommuting CMC nucleus	6 states	Properness
Structural self-conscious CMC nucleus	4 states	RR + indexical binding
Causal self-consciousness	5 states	Matched self-model on/off witness
Structural proleptic self-consciousness	4 states	Branch-complete future-self model + naturality
Causal proleptic self-consciousness	5 states	Matched future-self-model witness
Structural self-authorship	4 states	PSC + owned authorization; causal efficacy may be stipulated structurally
Operational causal self-authorship	5 states	Matched authorial-evaluation roots
Structural diachronic reciprocal authorship	4 states	Future standing + provenance + prospective ratifiability
Operational causal DRA	5 states	Matched roots differing only in modeled future ratifiability

These are minima relative to the conventions stated in the text. Changing what must be internally witnessed, whether semantic labels can share a state, or whether causal effects may be stipulated over uninstantiated counterfactuals can change the counts.

Appendix B. Dependency map

The following schematic summarizes the main conceptual dependencies established or proposed in the paper.



$$\begin{array}{c} \text{DRA} \quad -- (+ \text{ HAC}) --> \text{RDRA} \\ | \\ \text{K_DA} = (\text{N}, \text{I}, \text{A}, \text{P}, \text{U}) \end{array}$$

Separately, whole-monad first-person standing has the dependency

Monadic ontology + TV + SND -> diachronic non-haecceitistic standing,

while extending that result to internal wants requires RSI/LRS, and extending formal submonadhood to phenomenal subjectivity requires CSR. These are precisely the locations at which substantive bridge principles enter.

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End of research note