Possible Worlds of Interacting Husserlian Monads

Toward a Panprotopsychist Process Algebra of Subject Formation, Fusion, Fission, Overminds, and Collective Imagination

Abstract

This paper develops a speculative formal research program for an interacting monadology combining three ideas: panprotopsychism, quantum-process individuation, and a broadly Husserlian phenomenology of mature subjects. The motivating physical picture is inspired by Markopoulou's *Quantum Causal Histories*, in which finite-dimensional Hilbert spaces are associated with events or causal edges and local transformations compose along a directed causal network. In particular, Markopoulou notes that one may place Hilbert spaces on edges and treat events as transformations, and that trivalent structures naturally permit two-to-one and one-to-two moves. We reinterpret such moves as a formal environment in which monadic persistence, absorption, fusion, fission, birth, and death can be studied.

The central proposal is that a mature mindlike monad should not be identified with an instantaneous quantum tensor factor. It is better modeled as a persistent, state-bearing temporal process that may migrate through changing physical factorizations while maintaining a Husserlian organization involving retention, present awareness, protention, intentionality, horizons, object-synthesis, and an ego-pole. We then consider a category of possible psychophysical worlds, each characterized by its own laws governing the interactions and persistence of such monads.

A sequence of finite toy models is developed. These distinguish phenomenal copying from phenomenal sharing, overmind formation from fusion, psychological descent from numerical identity, and synchronic supervenience from diachronic constitution. A Boolean model shows that an emergent overmind can correspond to an autonomous quotient dynamics while fusion more naturally resembles projection onto a retract. Adding a collective memory produces a higher-order process whose state cannot be reconstructed from the instantaneous states of its constituents and which can nevertheless participate in reversible global dynamics. Enriching this process with intentionality yields higher-order noemata, self-awareness, nested societies of minds, novel collective intentions, and collective imagination. The resulting framework suggests that advanced subjecthood may consist not merely in integrated information but in the temporally self-maintaining production of an expanding space of intentional possibilities.

The paper concludes with a program for formalization: development of an intentional process algebra, classification of finite psychophysical worlds, rigorous criteria for subject emergence, quantum-channel models with memory, categorical treatment of nested subjecthood, and eventual comparison with candidate physical theories.

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1. Introduction

Panpsychist and panprotopsychist theories face an immediate structural problem. Even if the intrinsic nature of fundamental physical reality is somehow proto-phenomenal, one still has to explain the existence of enduring, unified, intentional subjects. How can simpler entities participate in, be absorbed by, or give rise to a mind? Under what conditions can minds fuse or divide? Can subjects exist at several constitutive levels at once? Can a higher-order subject possess experiences, intentions, or concepts unavailable to its constituent subjects?

The present project approaches these questions indirectly. Rather than attempting immediately to reconstruct human consciousness from known physics and biology, it asks a more abstract modal question:

> What kinds of worlds containing interacting Husserlian monads are coherent at all?

The strategy is analogous to the use of toy models in mathematical physics. We construct deliberately artificial "psychophysical worlds" whose laws need bear little resemblance to terrestrial biology. Their purpose is not empirical realism but conceptual separation. Human experience bundles together many properties—one body, one ego, one phenomenal field, one memory stream, one causal locus—that may not be metaphysically inseparable. Toy monadologies allow us to vary these independently.

Three ingredients organize the investigation.

First, the ontology is **panprotopsychist** rather than straightforwardly panpsychist. Fundamental monads need not themselves be subjects. They possess an intrinsic nature that may participate in phenomenality under suitable organization.

Second, monadic individuation is modeled using ideas suggested by **quantum process structure** rather than by classical substance metaphysics. Markopoulou's quantum causal histories provide an important inspiration: local state spaces are connected through causal transformations, and the entire history need not itself possess a single Hilbert space.

Third, mature mindlike monads are given a **Husserlian phenomenological organization**. At minimum, a subject has a present embedded in retention and protention; richer subjects possess intentional acts, noematic objects, horizons, habitualities, self-world differentiation, and an ego-pole organizing the stream as belonging to one perspective.

The result is not a finished metaphysical theory. It is better understood as the beginnings of a **space of theories**.

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2. From quantum events to persistent monads

2.1 Markopoulou's causal-process inspiration

In *Quantum Causal Histories*, Markopoulou begins with a causal set and associates finite-dimensional Hilbert spaces with events. Acausal sets carry tensor-product Hilbert spaces, and related acausal sets are joined by evolution operators.

A particularly suggestive reformulation puts Hilbert spaces on causal edges and evolution operators at events. Markopoulou motivates this by saying that events represent change and therefore are naturally associated with operators. Her trivalent examples include local structures with two incoming and one outgoing edge or one incoming and two outgoing edges.

These structures can be reinterpreted abstractly as:

$$A \otimes B \to C$$

or

$$C \to A \otimes B.$$

Nothing in Markopoulou's paper identifies such processes with minds; that is our speculative extension. But the formal architecture provides a natural arena for asking how individuated state-bearing systems can persist, merge, divide, or be replaced.

2.2 Monadic stages versus monadic continuants

The first major conceptual revision is that a mature monad should not be identified with an instantaneous tensor factor.

Instead, distinguish:

$$\boxed{\text{monadic stage}}$$

from

$$\boxed{\text{persistent monad.}}$$

A monadic stage is an approximately individuated physical/informational subsystem at one stage of a process. A persistent monad is an equivalence class or thread of stages related by an identity-preserving transport:

$$M_0 \Longrightarrow M_1 \Longrightarrow M_2 \Longrightarrow \cdots .$$

The underlying physical substrate may change radically along this thread. A persistent mind can incorporate simple monads, emit them, replace physical constituents, or change its effective factorization without ceasing to be the same subject.

This suggests that personal identity resides less in a fixed material factor than in a continuing organized process.

2.3 Identity and coarse-graining

Markopoulou imposes the composition law

$$E_{bc}E_{ab} = E_{ac},$$

which gives rise to directed coarse-graining invariance.

An analogous requirement is natural for monadic identity. If

$$T_{ab} : M_a \to M_b$$

and

$$T_{bc} : M_b \to M_c$$

are identity-preserving transports, then

$$\boxed{T_{bc} \circ T_{ab} \simeq T_{ac}.}$$

Identity should therefore be **coarse-graining invariant**. Whether one describes a subject's intervening history using millions of micro-events or one effective transition should not determine whether the same subject survives.

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3. Fusion, absorption, fission, birth, and death

A major early result of the project was the realization that several superficially similar processes must be kept distinct.

3.1 Absorption

Suppose

$$M + a \to M'$$

with

$$M \equiv M'.$$

Here the simple monad a is absorbed while M persists.

This need not be a fusion of two subjects. The proto-monad may cease to possess independent individuation while its intrinsic properties become incorporated into the continuing structure of M.

3.2 Emission

Conversely,

$$M \to M' + a$$

may preserve

$$M \equiv M'$$

while a new simple monad a begins.

Thus creation and destruction of simple monads can occur beneath the persistence of a mature mind.

3.3 Genuine fusion

If two full subjects undergo

$$A + B \to C,$$

ordinary numerical identity prevents us from asserting both

$$A = C$$

and

$$B = C,$$

unless $A = B$.

Thus genuine fusion may involve the death of both predecessors and the birth of a successor psychologically descended from each:

$$A \rightsquigarrow C, \qquad B \rightsquigarrow C.$$

3.4 Genuine fission

Likewise,

$$C \to A + B$$

cannot generally satisfy

$$A = C = B.$$

Both successors may inherit memories, habitualities, and a first-person sense of continuity without both being numerically identical to the predecessor.

We therefore introduced a weaker descent relation:

$$C \rightsquigarrow A, \qquad C \rightsquigarrow B.$$

The lesson is:

$$\boxed{\text{psychological continuity } /} \qquad = \text{numerical iden}$$

3.5 Identity strands

A mature monad's identity may therefore be represented as a nonbranching strand embedded inside a branching and merging causal network.

At a vertex an identity strand may:

$$\text{continue}, \qquad \text{begin}, \qquad \text{end},$$

while the physical causal structure can freely branch and merge.

This gives distinct formal notions of birth, death, fission, fusion, absorption, and emission.

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4. A category of possible psychophysical worlds

Rather than stipulating one universal set of monadic laws, we proposed studying a family of possible worlds.

Let

$$W = (\mathcal{M}_W, \otimes, \mathcal{L}_W, \mathcal{P}_W, \equiv_W)$$

denote a psychophysical world.

Here:

- $\mathcal{M}_W$ is its collection of possible monadic states;
- $\otimes$ represents coexistence or parallel composition;
- $\mathcal{L}_W$ gives permitted interactions;
- $\mathcal{P}_W$ gives psychophysical realization laws;
- $\equiv_W$ gives persistence or identity.

A process can have the form

$$A_1 \otimes \cdots \otimes A_m \longrightarrow B_1 \otimes \cdots \otimes B_n.$$

This suggests a symmetric monoidal category, PROP, properad, or related many-input/many-output formalism.

The worlds themselves can then form a higher category

$$\mathbf{PsychWorld},$$

with morphisms interpreted as:

- embeddings;
- simulations;
- coarse-grainings;
- forgetful maps;
- equivalences between descriptions.

The long-term ambition is a **moduli space of monadologies**: a classification of coherent forms of psychophysical law.

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5. Minimal Husserlian structure

A mature mind-monad was given a schematic state

$$M = (E, R, I, P, \mathcal{N}, \mathcal{V}, \Sigma, H).$$

The symbols represent:

- E: ego-pole or perspectival center;
- R: retention;
- I: current phenomenal/intentional field;
- P: protention;
- $\mathcal{N}$: noematic structures;
- $\mathcal{V}$: noetic modes;
- Σ: synthesis of object identity;
- H: intentional horizons.

The minimal temporal structure is:

$$R_t - I_t - P_t.$$

Persistence requires some form of

$$I_t \hookrightarrow R_{t+1}.$$

Thus the immediately preceding present remains incorporated into the succeeding moment as retained past.

This suggested one of the project's recurring hypotheses:

$$\boxed{\text{phenomenal subjecthood may depend more deeply on temporal synthesis than on static integration alone.}}$$

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6. Toy worlds separating forms of mental interaction

A series of finite worlds was constructed to separate notions normally conflated.

6.1 Assimilation world

$$H_E(r, i, p) \otimes m_q \to H_E(i, q, q).$$

The proto-monad's intrinsic quality becomes part of the mind's phenomenal state while the ego-pole persists.

This separates phenomenal incorporation from subject fusion.

6.2 Telepathic-copy world

A noematic or phenomenal content is copied:

$$T_N^{A \to B}(n_A) = n'_B.$$

Two subjects can therefore possess qualitatively identical content without becoming one subject.

6.3 Shared-token world

A stronger law permits one phenomenal token to belong simultaneously to two ego-streams.

Then:

$$\text{same phenomenal token}$$

need not imply

$$\text{same subject}.$$

This tests whether phenomenal-token ownership is metaphysically exclusive.

6.4 Overmind world

Two persistent lower subjects generate a third higher-order subject:

$$A + B \to A + B + C.$$

The overmind's state depends on relations between A and B that neither constituent needs to experience individually.

This separates higher-order subject formation from fusion.

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7. The intentional process algebra

The next step was to replace bare phenomenal bits by explicitly intentional structure.

An intentional episode can be represented by

$$\iota = (\nu, n, \pi, h),$$

where:

- ν is noetic mode;
- n is noema;
- π is positional/doxic attitude;
- h is horizon.

A grounding relation

$$\rho(n) = X$$

connects a noema to an external object, while

$$\rho(n) = \bot$$

allows fiction, hallucination, or imagination.

A tentative universal intentional algebra HInt was proposed with generators including

$$\mathrm{Present}, \mathrm{Intend}, \mathrm{Attend}, \mathrm{Retain}, \mathrm{Protend}, \mathrm{Fulfill}, T_N, T_\nu, T_H, \mathrm{Recognize}, \mathrm{Joint}, \mathrm{Fuse}, \mathrm{Fiss}\,.$$

Different worlds realize different subsets and compositions of these operations.

One important decomposition was:

$$\text{noema transfer} \;/\; \text{noesis transfer} \not= \text{horizon transfer}.$$

Thus one subject might receive another's object-as-intended without sharing the same attitude toward it.

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8. Conditions for the emergence of a new subject

We then asked what distinguishes a genuine new subject from a complicated relation.

A hierarchy of candidate conditions emerged.

8.1 Unified present

There must be some higher-order state

$$I_C$$

not reducible merely to the ordered pair

$$(I_A, I_B).$$

8.2 Retentional closure

$$I_C(t) \to R_C(t+1).$$

The collective present becomes the past of the same collective stream.

8.3 Protentional closure

$$P_C(t)$$

must organize the collective's own future possibilities.

8.4 Perspectival closure

There must exist a higher-order "for-whom" structure

$$E_C.$$

8.5 Dynamical closure

The collective state must possess autonomous predictive relevance:

$$\Gamma_C(t) \to \Gamma_C(t+1).$$

8.6 Nonfactorizability

The higher-order process must not reduce without remainder to two independent lower processes.

This led to the working hypothesis:

$$\boxed{\text{A new subject forms when a nonfactorizable intentional process develops its own temporally closed perspectival or}}$$

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9. The tiny Boolean model

The most concrete part of the investigation began with two binary lower subjects:

$$a_t, b_t \in \{0, 1\}.$$

Define a candidate collective variable:

$$c_t = a_t \oplus b_t.$$

The joint microscopic state space is

$$X = \{00, 01, 10, 11\}.$$

Let

$$p(a, b) = a \oplus b.$$

A collective autonomous dynamics exists when there is a map

$$\bar{T}$$

such that

$$\boxed{pT = \bar{T}p.}$$

This is the condition for c to define a dynamical quotient.

The conceptual lesson was more important than the details:

$$\boxed{\text{overmind emergence naturally resembles formation of an autonomous factor or quotient.}}$$

By contrast, fusion was more naturally represented by a projection that destroys lower-level distinctions.

For example,

$$(a, b) \mapsto (0, a \oplus b)$$

projects four microstates onto two surviving collective states. With

$$i(c) = (0, c),$$

one has

$$p \circ i = 1_C$$

and

$$e = i \circ p, \qquad e^2 = e.$$

Hence:

$$\boxed{\text{overmind formation} \sim \text{quotient/factor, fusion} \quad \sim \text{projection/retract.}}$$

This refined an earlier idea that all subject emergence should be modeled by split idempotents.

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10. Diachronic emergence

We next added a collective memory bit

$$m_t$$

with

$$m_{t+1} = c_t.$$

Take

$$a_{t+1} = a_t \oplus m_t, \qquad b_{t+1} = b_t.$$

Then

$$c_{t+1} = c_t \oplus m_t.$$

The collective state is

$$C_t = (c_t, m_t).$$

The key property is that

$$m_t$$

cannot in general be reconstructed from the simultaneous lower state

$$(a_t, b_t).$$

Two worlds can have identical current A and B states but different collective memories and therefore different futures.

This yields the important distinction:

$$\boxed{\text{synchronic supervenience } /} \qquad = \text{diachronic}$$

The higher-order process is partly constituted by its own history.

Remarkably, the total finite-state dynamics can remain reversible. Thus a higher-order state need not arise by destroying information in its lower-level substrate.

This suggested replacing the picture

$$\text{mind} \sim \text{instantaneous tensor factor}$$

with something closer to

$$\boxed{\text{mind} \sim \text{stateful channel or persistent process carrying memory through successive interactions.}}$$

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11. The intentional overmind

The collective process was then given genuinely higher-order noemata.

Define

$$J_i = \mathrm{Joint}(A, B, X_i),$$

meaning roughly:

> X_i as jointly intended by A and B.

Neither lower subject needs to possess J_i.

The overmind can retain a joint noema during temporary disruption:

$$R_C = J_i,$$

while protending its restoration:

$$P_C = J_i^*.$$

If subsequent dynamics restores joint attention, one obtains:

$$J_i^* \xrightarrow{\text{fulfillment}} J_i.$$

If the wrong joint object appears, there is disconfirmation.

Thus the overmind possesses an intentional arc:

$$\boxed{\text{presentation} \to \text{loss} \to \text{retention} \to \text{protention} \to \text{fulfillment/disconfirmation.}}$$

This is a much stronger candidate for subjecthood than a mere macro-variable.

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12. Self-consciousness of the overmind

The overmind was then given an intentional ontology containing:

$$X_0, X_1, A, B, J_0, J_1, D,$$

where D represents divergence of the lower subjects.

A complete inner representation of C would generate a regress:

$$C \to (C \to (C \to \cdots)).$$

To avoid this, we introduced an indexical self-noema

$$\sigma_C,$$

roughly meaning:

> this very continuing we-subject.

The crucial explanatory order is:

$$\boxed{\text{temporally integrated stream} \to \text{stable ownership pole} \to \text{self-noema.}}$$

The subject does not come into existence because it contains a symbol referring to itself.

Rather, an already existing temporally unified subject acquires the ability to take its own ego-pole as an intentional object.

This permits explicit self-judgments such as:

$$\text{SameSubject}(\sigma_C^{t_0}, \sigma_C^{t_1}),$$

and intentional representations of membership:

$$\text{Member}(A, C), \qquad \text{Member}(B, C).$$

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13. Societies of nested minds

Once a constituent can intentionally represent the higher subject of which it is a part, a new form of recursive society appears.

Four relations must be kept distinct:

$$\equiv \quad \text{identity},$$

$$\prec \quad \text{constitution},$$

$$\rightsquigarrow \quad \text{intentional reference},$$

$$\rightarrow \quad \text{causal influence}.$$

Thus one may coherently have:

$$A \prec C,$$

$$A \rightsquigarrow C,$$

$$C \rightsquigarrow A.$$

A subject can therefore be aware of the higher subject it helps constitute.

Several forms of collective organization follow:

- opaque overminds;
- transparent overminds;
- reflexive overminds;
- participatory overminds;
- overlapping higher subjects;
- changing membership;
- nested hierarchies or holarchies.

A higher subject can possess conscious states unavailable to lower subjects:

$$J_C \in \Phi_C, \qquad J_C \notin \Phi_A, \Phi_B.$$

Conversely, a lower subject can possess a conscious state that influences C without appearing in C's phenomenal field:

$$p_A \in \Phi_A, \qquad p_A \notin \Phi_C.$$

Thus:

$$\boxed{\text{consciousness is level-relative.}}$$

A higher mind can therefore have something analogous to an unconscious composed partly of processes that are themselves conscious at a lower constitutive level.

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14. Emergent collective intentions

The next question was whether C can want something that none of its members wants.

Suppose A and B both attend to X_0 and both locally intend to remain there.

C can constitute the higher-order noema

$$R_0 = \mathrm{Redundant}(A, B, X_0),$$

and generate the intention

$$K_C = \mathrm{Diversify}\,.$$

Its goal set is

$$G_C = \{(0,1), (1,0)\}.$$

This condition is irreducibly relational: it is not a property of A alone or B alone.

Thus:

$$\boxed{\text{a collective subject can possess a goal that belongs to no constituent individually.}}$$

C can articulate this goal into complementary roles:

$$\mathsf{Art}(K_C) = (\mathrm{Stay}_A, \mathrm{Switch}_B).$$

Once lower-level autonomy is restored, members can accept, reject, or modify the proposed roles.

This allows genuine internal politics:

$$\text{C wants } K,$$

while

$$A \text{ or } B \text{ rejects } K.$$

Negotiation can then generate a new intention that none of the parties had previously possessed.

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15. Collective imagination

The final major development was to allow C to create new noemata.

Suppose A and B have intentionally impoverished vocabularies:

$$\mathcal{N}_A = \{A_0, A_1\},$$
$$\mathcal{N}_B = \{B_0, B_1\}.$$

C possesses constructors such as

$$\mathrm{Pair}, \mathrm{Seq}, \mathrm{Loop}, \mathrm{Abstract}\,.$$

It can construct:

$$D_{01} = \mathrm{Pair}(A_0, B_1),$$
$$D_{10} = \mathrm{Pair}(A_1, B_0),$$

and then:

$$Z = \mathrm{Loop}\left(\mathrm{Seq}(D_{01}, D_{10})\right).$$

Z represents the possible temporal pattern:

$$01 \to 10 \to 01 \to 10 \to \cdots\,.$$

Neither constituent can represent Z.

Initially:

$$\rho(Z) = \bot.$$

C therefore intends it under imagination:

$$(\mathrm{imagine}, Z).$$

The same noema can subsequently undergo:

$$\mathrm{imagine}(Z) \to \mathrm{desire}(Z) \to \mathrm{will}(Z).$$

C then articulates the project into local instructions that A and B can execute without themselves understanding the global noema.

Thus:

a higher subject can imagine a possibility unavailable to all constituents and realize it through them.

A less ad hoc model allows C first to abstract a pattern from its own retained history:

$$D_{01}, D_{10}, D_{01}, D_{10} \to \mathsf{ALT},$$

then detach the pattern from actuality:

$$\mathsf{ALT} \to \mathsf{ALT}^{*}.$$

This gives the sequence:

$$\boxed{\text{synthesis} \to \text{abstraction} \to \text{free variation} \to \text{imagination} \to \text{project.}}$$

Collective creativity thus becomes an expansion of the higher subject's own intentional state space:

$$\mathcal{N}_C(t) \subsetneq \mathcal{N}_C(t+1).$$

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16. Main conceptual results

The exploration has produced a number of tentative but relatively sharp conclusions.

16.1 Mature monads are better modeled as processes than instantaneous factors

The appropriate ontological unit appears to be a persistent state-bearing process with memory, not a fixed tensor factor.

16.2 Persistence and material composition can come apart

A subject may survive extensive or complete replacement of constituent proto-monads.

16.3 Fusion and overmind formation are distinct

Overmind formation can preserve constituent subjects:

$$A + B \to A + B + C.$$

Fusion removes their independent persistence:

$$A + B \to C.$$

Mathematically, the former resembles formation of a quotient/factor, while the latter resembles projection onto a surviving retract.

16.4 Identity is stricter than continuity

Fission can preserve memory, personality, and apparent first-person continuity while failing to preserve unique numerical identity.

16.5 Subjecthood appears deeply diachronic

A higher-order subject becomes much more plausible when some of its present state is inherited from its own preceding state rather than reconstructed from its instantaneous substrate.

16.6 Higher-order intentionality can be genuinely novel

A higher subject may possess noemata, beliefs, goals, and possibilities unavailable to every constituent.

16.7 Consciousness need not be transparent across constitutive levels

Lower-level conscious states can be unconscious for a higher subject, while higher-level conscious states can be inaccessible to lower subjects.

16.8 Collective agency can conflict with constituent agency

A higher subject may possess a will not reducible to current lower-level preferences.

16.9 Collective imagination may enlarge the space of possible intentional states

An advanced monad need not merely move through a fixed state space. It can generate new representational forms:

$$S_C(t) \hookrightarrow S_C(t+1).$$

This may be one of the strongest signatures of advanced mindhood in the framework.

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17. A provisional architecture

The emerging ontology can be summarized as:

$$\boxed{\text{proto-monadic intrinsic reality} \downarrow \text{temporally persistent monadic processes} \downarrow \text{Husserlian intentional monads} \downarrow}$$

The central conjecture may be expressed as:

Advanced monadic subjecthood consists not merely in integrated state, but in a nonfactorizable, ter

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18. Next steps

The project is now developed enough that the next phase should become more formal and less discursive.

18.1 Formalize the intentional algebra

The proposed generators

$$\mathrm{Present}, \mathrm{Attend}, \mathrm{Retain}, \mathrm{Protend}, \mathrm{Fulfill}, T_N, T_\nu, T_H, \mathrm{Recognize}, \mathrm{Joint}, \mathrm{Fuse}, \mathrm{Fiss}$$

should be given explicit types, composition rules, and coherence equations.

One goal would be a finite presentation of an intentional PROP, operad, or symmetric monoidal category

$$\mathsf{HInt}.$$

The immediate task is to distinguish:

1. type-theoretically impossible compositions;
2. universally coherent compositions;
3. compositions whose admissibility depends on the psychophysical world.

18.2 Classify finite psychophysical worlds computationally

The Boolean examples should be expanded systematically.

For finite state spaces one can enumerate:

- microscopic transition laws;
- quotient dynamics;
- persistent memories;
- fusion and fission maps;
- subject-support relations;
- intentional constructors;
- self-reference structures.

The goal would be a genuine **phase diagram of finite monadologies**.

Possible invariants include:

$$(\mathrm{privacy}, \mathrm{fusion}, \mathrm{fission}, \mathrm{nesting}, \mathrm{memory}, \mathrm{selfhood}, \mathrm{generativity}).$$

18.3 Formalize subject emergence as a family of increasingly strong criteria

Rather than impose one definition, compare:

$$S_1 = \text{collective definability},$$

$$S_2 = S_1 + \text{autonomous dynamics},$$

$$S_3 = S_2 + \text{retention},$$

$$S_4 = S_3 + \text{protention},$$

$$S_5 = S_4 + \text{perspectival closure},$$

$$S_6 = S_5 + \text{causal self-maintenance},$$

$$S_7 = S_6 + \text{intentional generativity}.$$

Toy worlds should then be constructed that satisfy some but not others.

This may reveal which conditions are genuinely constitutive of subjecthood and which merely characterize advanced cognition.

18.4 Develop a quantum-channel version

The finite-state models should eventually be translated into quantum-process language.

A persistent monad might be represented by a channel with internal memory:

$$\mathcal{C} : \mathcal{H}_{\text{in}} \otimes \mathcal{H}_M \to \mathcal{H}_{\text{out}} \otimes \mathcal{H}_M.$$

The carried state

$$\rho_M(t)$$

would encode diachronic monadic organization.

This would allow a rigorous comparison between:

- instantaneous tensor-factor individuation;
- quantum memory;
- entanglement;
- decoherence;
- persistent logical subsystems;
- higher-order process structure.

The point would not initially be to model brains, but to determine whether the toy distinctions survive genuine quantum mechanics.

18.5 Study fusion and fission with quantum information preserved

A key question is whether one can have globally reversible quantum evolution while monadic individuation changes:

$$A + B \to C$$

or

$$C \to A + B.$$

The information need not disappear even if subject identities do.

Quantum error-correcting codes, subsystem codes, quantum combs, and memory channels may provide useful mathematical analogies.

18.6 Develop the category of psychophysical worlds

The proposed category

$$\mathbf{PsychWorld}$$

should be given a precise definition.

Important morphisms include:

$$\text{embedding}, \quad \text{simulation}, \quad \text{coarse-graining}, \quad \text{forgetful map}, \quad \text{equivalence}.$$

A higher-categorical treatment may eventually be appropriate because there may be distinct but equivalent translations between worlds.

The long-range goal would be a **moduli theory of monadologies**.

18.7 Formalize nested societies using a double category or higher operad

Nested subjects involve at least two distinct relations:

$$\text{intentional interaction}$$

and

$$\text{constitution}.$$

A double category is therefore a natural candidate:

$$A \xrightarrow{\text{intentional}} B \Downarrow \qquad \Downarrow C \xrightarrow{\text{intentional}} D.$$

Vertical arrows would encode constitution, horizontal arrows interaction, and squares their compatibility.

A multicategorical extension would allow many constituent subjects to jointly constitute one higher subject.

18.8 Investigate emergent normativity

Once a subject possesses:

$$\sigma_C,$$

a representation of its own continuation, it can form self-directed protentions:

$$\mathrm{Preserve}(C).$$

This produces a primitive distinction between states that fulfill and frustrate self-directed intentionality.

One should investigate whether this yields a general route:

$$\text{self-representation} + \text{protention} + \text{agency} \to \text{endogenous normativity}.$$

This may connect subjecthood with primitive value.

18.9 Study collective creativity more seriously

The imagination model suggests that intentional state spaces themselves can grow.

A formal theory should distinguish:

$$\text{composition}, \quad \text{abstraction}, \quad \text{analogy}, \quad \text{negation}, \quad \text{counterfactual variation}, \quad \text{concept creation}.$$

One could then ask whether particular algebraic structures guarantee open-ended growth of

$$\mathcal{N}_C.$$

This would turn the theory from a dynamics on a fixed state space into a theory of **state-space generation**.

18.10 Only later compare with our world

The program should resist premature neurophysical identification.

The immediate question remains:

$$\text{What psychophysical organizations are coherent?}$$

Only after this possibility space is better understood should one ask:

$$\text{Which structures, if any, are instantiated by actual quantum systems, organisms, brains, or societies?}$$

The distinction between modal exploration and empirical hypothesis is essential.

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19. Conclusion

The project began with a relatively simple image: a panprotopsychist universe composed of interacting quantum-like monads, some of which develop a roughly Husserlian phenomenal organization.

Following the logic of interaction transformed that image considerably.

A mature monad came to look less like a permanent substance or tensor factor than a **diachronically persistent process**. Fusion and fission forced a separation between numerical identity and psychological descent. Toy worlds showed that shared experience, synchronization, fusion, and collective subjecthood need not coincide. A Boolean model distinguished overmind formation from fusion and showed how a collective process can possess autonomous dynamics. Adding memory produced a higher-order state irreducible to the instantaneous states of the constituents. Adding intentionality produced relational noemata, fulfillment, surprise, and a higher-order world. Adding self-reference produced a we-pole and a self-noema. Nested subjects generated societies of minds. Those societies could acquire intentions not possessed by their members, and eventually imagine possibilities that no constituent could even represent.

The resulting ontology is therefore not simply a panpsychist combination theory. It is a proposed **dynamics of possible subjecthood**.

Its characteristic entities can:

persist, absorb, emit, fuse, fission, nest, recognize, intend, imagine, create new intentional forms

The resulting research question is correspondingly broader:

> What is the mathematical space of possible laws governing the birth, persistence, interaction, self-organization, ar

The hope is that a systematic theory of such possible worlds may eventually clarify both the combination problem of panprotopsychism and the deeper question of what a subject is. Rather than beginning with the assumption that minds must have the architecture familiar from terrestrial organisms, interacting Husserlian monadology invites us first to explore the much larger space of what mindlike existence could be.